Problem. Let A be a point outside a circle ω, centered at O, and let B and C be points on each of the tangent segments from A to ω, such that the line segment BC is also tangent to ω. Let Ω be the circumcircle of triangle ABC, centered at P. The lines OB and OC intersect Ω again at D and E, respectively. The lines DP and EP intersect AC and AB, respectively, at Q and R. Prove that the circumcircles of $\triangle ABC$ and $\triangle PQR$ are tangent to each other.

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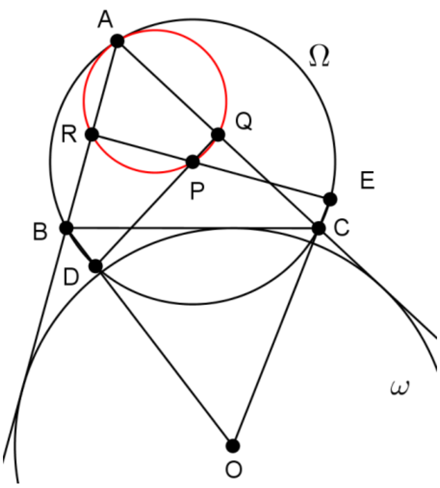
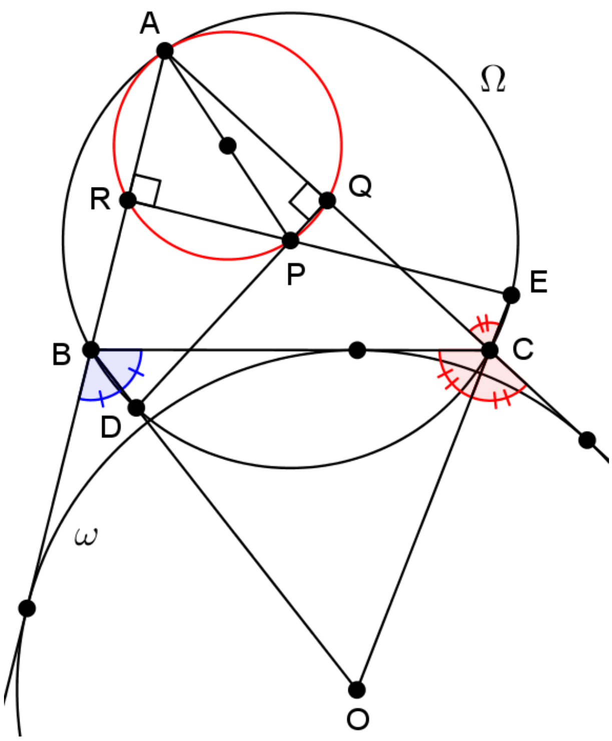


Proof (Stefan Lozanovski)**.**
Since BA and BC are tangents to ω and O is its center, by symmetry we get that BO is the angle bisector of the exterior angle at the vertex B in triangle ABC. Therefore, $\angle DBC = \frac{\alpha+\gamma}{2}$, i.e. D is the midpoint of the arc $\widehat{ABC}$. Since P is the center of Ω, we ge that DP is the side bisector of AC, i.e. $\angle PQA = 90°$. Similarly, E is the midpoint of arc $\widehat{ACB}$ and $\angle PRA = 90°$.



Therefore $A \in PQR$. By Thales' Theorem, the center of this circle is the midpoint of the diameter AP. Finally, since A lies on both (ABC) and (PQR), and since A is collinear with their centers, we get that these circles are tangent to each other ■