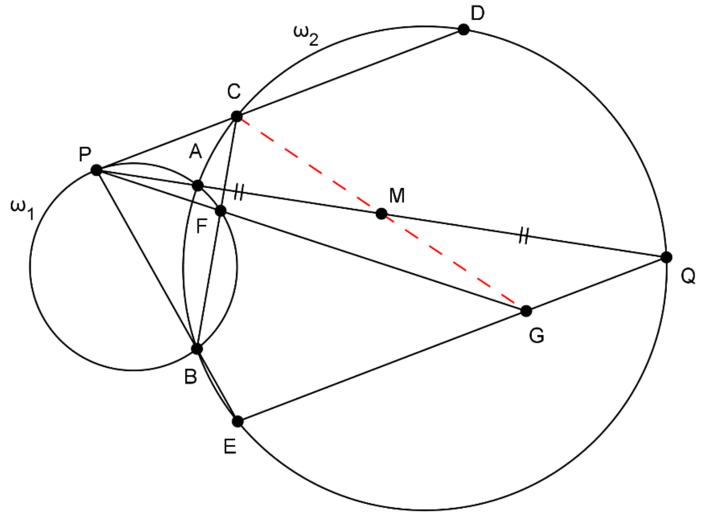
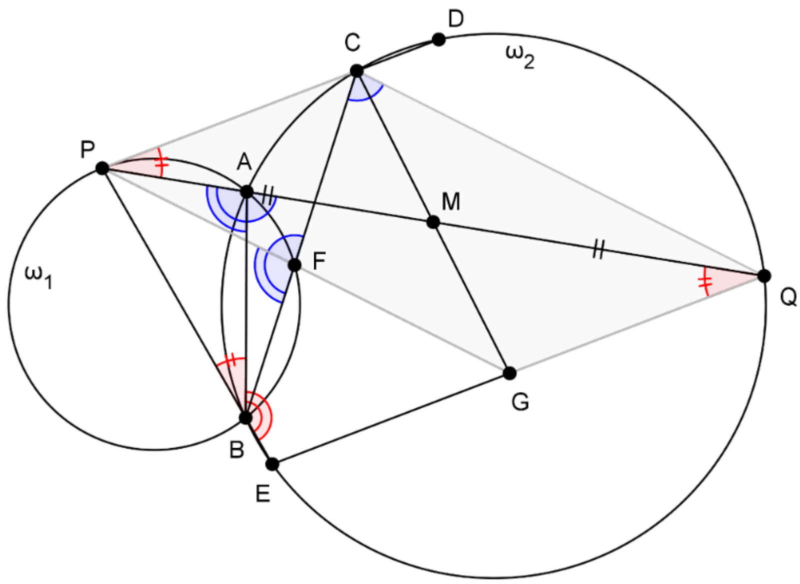
Problem. Two circles ω_1 and ω_2 intersect at A and B ($r_1 < r_2$). Let P be a point on ω_1 (P is outside of ω_2 and $\overline{PA} < \overline{PB}$), such that the tangent to ω_1 at P intersect ω_2 at two distinct points C and D (C is between P and D). Let PA and PB intersect ω_2 again at Q and E, respectively (B is between P and E) and let M be the midpoint of PQ. The line BC intersects ω_1 again at F and PF intersects QE at G. Prove that C, M and G are collinear.

Proposed by: Stefan Lozanovski, Macedonia



Proof (Stefan Lozanovski).
We will prove that $PCQG$ is a parallelogram.



$$\angle CPQ \equiv \angle CPA = \angle PBA = 180° - \angle ABE = \angle AQE \equiv \angle PQG$$
$$\therefore PC \parallel GQ$$

$$\angle PFC = 180° - \angle PFB = 180° - \angle PAB = \angle BAQ = \angle BCQ \equiv \angle FCQ$$
$$\therefore PF \parallel CQ, \text{ i.e. } PG \parallel CQ$$

Therefore, $PCQG$ is a parallelogram. Because its diagonals bisect and because M is the midpoint of the diagonal PQ, then M must lie on the other diagonal CG, i.e. C, M and G are collinear ■

Remark 1. If a slightly easier version of this problem is needed, the problem statement can be "Prove that $\overline{CM} = \overline{MG}$ ". In this way, it is more obvious for the student that they need to prove that $PCQG$ is a parallelogram.

Remark 2. The conditions in parentheses of the problem statement are not needed for the problem to be true. They are put there so all the students would get the same configuration and they wouldn't have to care for different configurations when writing their proofs.