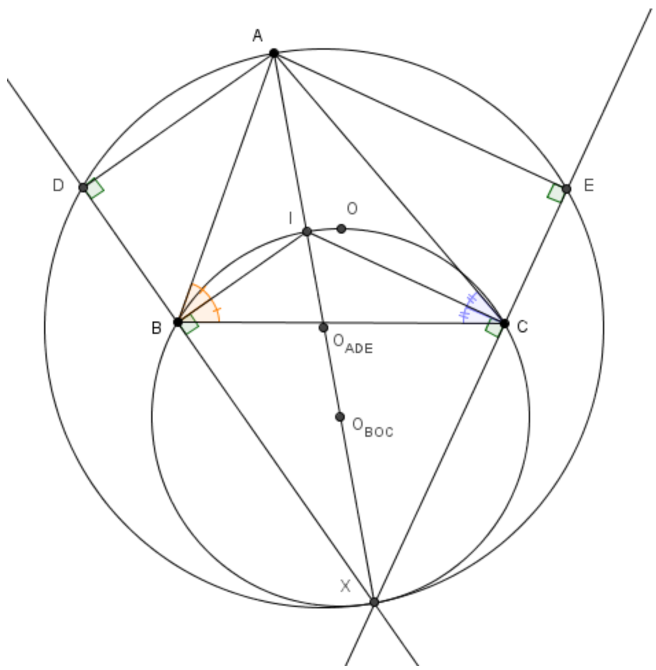
(Stefan Lozanovski) Let ABC be a triangle with $\angle BAC = 60^\circ$. Let D and E be the feet of the perpendiculars from A to the external angle bisectors at the vertices B and C in $\triangle ABC$, respectively. Let O be the circumcenter of $\triangle ABC$. Prove that the circumcircles of $\triangle ADE$ and $\triangle BOC$ are tangent to each other.

Proof. (Stefan Lozanovski) Let X be the intersection of the lines BD and CE. We will firstly prove that X lies on the circumcircles of both $\triangle ADE$ and $\triangle BOC$ and then we will prove that the centers of these circles and the point X are collinear, which is enough to prove that the circles are tangent to each other.

In this proof, we will use the notations (MNP) and O_{MNP} to denote the circumcircle and the circumcenter of a $\triangle MNP$, respectively.



$$\angle ADX + \angle AEX = 90^\circ + 90^\circ = 180^\circ$$

$$\therefore X \in (ADE),\ O_{ADE} \in AX \tag{1}$$

Let I be the incenter of $\triangle ABC$.

$$\angle IBX = \angle IBC + \angle CBX = \frac{\beta}{2} + \frac{180^\circ - \beta}{2} = 90^\circ$$

Similarly, $\angle ICX = 90^\circ$.

$$\angle IBX + \angle ICX = 90^\circ + 90^\circ = 180^\circ$$

$$\therefore X \in (BIC),\ O_{BIC} \in IX \tag{2}$$

Now, because $\alpha = 60°$, we have:

$\angle BOC = 2 \cdot \angle BAC = 2\alpha = 120°$

$\angle BIC = 180° - (\angle IBC + \angle ICB) = 180° - (\frac{\beta}{2} + \frac{\gamma}{2}) = 90° + \frac{\alpha}{2} = 120°$

$\therefore \angle BOC = \angle BIC$

$\therefore O \in (BIC)$

$\therefore (BIC) \equiv (BOC),\ O_{BIC} \equiv O_{BOC}$ (3)

It is well known that the internal angle bisector at A and the external angle bisectors at B and C are concurrent (at the A-excenter), so the points $A - I - X$ are collinear. Combining with (1), (2) and (3), we can conclude that the points $O_{ADE} - O_{BOC} - X$ are collinear. ■

Remark: In the statement of the problem, $\triangle BOC$ can be substituted with $\triangle BHC$, where H is the orthocenter of $\triangle ABC$. This is because H lies on the circumcirlce of $\triangle BOC$ ($\angle BHC = 180° - \alpha = 120° = \angle BOC$)